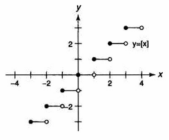
DO NOW

What do you know about the continuity of the following function?

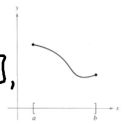


2.4 Continuity & One-Sided Limits - Day 2

Continuity on a Closed Interval:

A function f is continuous on the closed interval $[a,b]$, if it is continuous on the open interval (a,b) and

$\lim_{x \to a^+} f(x) = f(a)$ AND $\lim_{x \to b^-} f(x) = f(b)$



* What happens at the endpoints!

Theorem 2.11 Properties of Continuity

If the functions f and g are continuous at x = c, then the following combinations are continuous at x = c.

1. Sums: $f+g$
2. differences: $f-g$
3. products: $f \cdot g$
4. quotients: $\frac{f}{g}$ provided $g(x) \neq 0$
5. Scalar multiples: $b \cdot f$ where $b \in \mathbb{R}$

Functions that are continuous everywhere in their domain:

1. polynomials
2. rational (denom $\neq 0$)
3. radical
4. Trig functions ($\sin x, \cos x \ldots$)
5. Exponential and logarithmic

Theorem 2.12 Continuity of a Composite Function

If g is continuous at c and f is continuous at g(c):

then the composite function $f(g(c))$ or $f \circ g(c)$ is continuous at c.

Examples of composite functions:

$f(x) = \sin 3x$

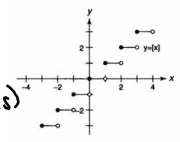
$f(x) = \sqrt{x^2+1}$

The Greatest Integer Function:

f(x) = [[x]] defined as [[x]] = greatest integer n such that $n \leq x$.

For example: [[2.5]] = 2 [[-2.5]] = -3

Discuss the continuity of this function.



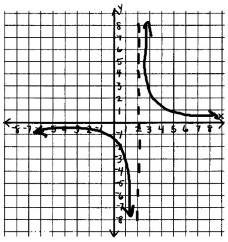
Not everywhere continuous (there are continuous intervals)

Nonremovable discontinuity at every integer

Limit at integers D.N.E.

Examples: Discuss the continuity of the function.

1. $f(x) = \frac{1}{x-2}$



$f(2) = D.N.E.$

Not everywhere continuous

NonRemovable discontinuity at $x=2$

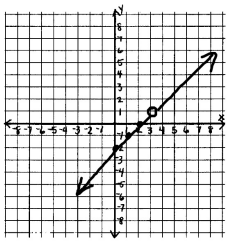
$\lim_{x \to 2} f(x) = D.N.E.$

* Is continuous over its domain

2. $f(x) = \frac{x^2 - 5x + 6}{x - 3}$

$f(x) = \frac{(x-2)(x-3)}{x-3}$

$f(x) = x - 2 \quad \boxed{x = 3}$

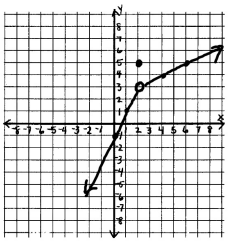


Not everywhere continuous

Removable discontinuity at $x=3$

$\lim_{x \to 3} f(x) = 1$

3. $f(x) = \begin{cases} 2x - 1, & x < 2 \\ 5, & x = 2 \\ \frac{1}{2}x + 2, & x > 2 \end{cases}$



Not everywhere continuous

Removable discontinuity at $x=2$

$\lim_{x \to 2} f(x) = 3$

$f(2) = 5$ * Can fill the hole by redefining

HOMEWORK

19-27 odd

pg 99 - 102; 29 - 32, 112

33-51 odd